

# Weighted ICP Algorithm for Alignment of Stars from Scanned Astronomical Photographic Plates

Alexander Marinov, Nadezhda Zlateva, Dimo Dimov and Delian Marinov

**Abstract:** *Given the coarse celestial coordinates of the centre of a plate scan and the field of view, we are looking for a mapping between the stars extracted from the image and the stars from a catalogue, where the stars from both sources are represented with their stellar magnitudes and coordinates, relatively to the image centre. In a previous work we demonstrated the appliance of Iterative Closest Point (ICP) algorithm for the alignment problem where stars were represented only by their geometrical coordinates. ICP leads to translation and rotation of the initial points - a correction required for the one set of stars to fit over the other. This paper extends the previous work by demonstrating significant improvement of ICP by using the stellar magnitudes as point weights. The improvement consists of great decrease of the iteration count until convergence, which helps in the case of more misaligned initial states. The essential aspects of the ICP method like noise tolerance of false or missing stars are still in charge.*

**Key words:** *Iterative Closest Point (ICP) algorithm, stellar alignment, weighted stellar alignment astronomical wide plate registration.*

## INTRODUCTION

The problem of stars alignment appears in cases when two or more sets of stars obtained from different sources must be registered in the same database or astronomical catalogue. The contemporary photographic instruments produce high quality images and provide high fidelity measurements characterizing the image as part of the sky. Nevertheless, there are a large number of photographic plates from the observatories' archives that suffer from the quality of the past instruments. The Institute of Astronomy at the Bulgarian Academy of Sciences holds a large archive of photographic plates - "Catalogue of Wide-Field Plate Archives" [9] enumerating over  $2 \cdot 10^6$  observations obtained since the end of 19th century from observatories worldwide. This historical heritage is highly important for an astronomer who studies the evolution of the sky map through the years. In the course of the digitization process the photographic plates are scanned and then formatted in FITS format. Together with the image data the FITS file also holds meta-information (FITS header) characterizing variety of image attributes. The FITS images must be supplied with specific attributes to be approved for registration in a Virtual Observatory [9] and the minimum consists of the image centre, field of view and in the case of wide-field plates - nonlinear distortion coefficients. The archived plates contain various kinds of defects, such as obscured surface areas due to physical damage, image noise-artefacts like the artificial réseau grid, lost information regarding some of the critical image parameters or completely "lost in space" or "orphan" plates.

This paper contributes to a project<sup>1</sup> aiming to automate the correction of FITS image header where some of the critical characterizing parameters are not supplied or not accurate. Using the approach of the ICP method we are able to find the translation and rotation necessary to align the stars extracted from an image [2] with the stars queried from a standard star catalogue. That alignment consists of minimizing the *distance* between star pairs for a certain star-to-star mapping. In [7] the algorithm was experimented by using Euclidian distance between geometrical points. Herein we will

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demonstrate the significant improvement of the algorithm performance and accuracy by extending the distance function to use assigned weights to the geometrical points.

For the application of the ICP method the coarse celestial coordinates of the image centre and the field of view must be initially given.

In the case of orphan plates where the image is completely lost the ICP method is not suitable. Rather, algorithms like [5] must be used prior the presented method, in order to supply ICP with more comfortable initial stage. In addition, the stars extractor instrument [2] should also be adapted to provide estimation of the stellar magnitudes.

The course of wide-field plate registration involves one or more of the following separate steps:

1. Scanning
2. Filtering the regular noise and artefacts from the input image
3. Extracting the stars centres and magnitudes
4. Searching the stars in a star catalogue to determine the image location on the sky [5]
5. Aligning the extracted stars with the set of stars from the catalogue [7]

Assuming the initial steps 1÷4 are already solved; we focus on resolving step 5.

## MODEL AND PROBLEM DEFINITION

We are considering the star maps from the image and from the catalogue as two point sets,  $A$  and  $B$ , in the Euclidian 2-dimensional space  $E^2$ :

$$A, B \subset E^2, \quad |A| = n, \quad |B| = m, \quad (3)$$

$$W_A : A \rightarrow \mathbf{R}, \quad W_B : B \rightarrow \mathbf{R}$$

Where the functions  $W_A$  and  $W_B$  represent the assigned weights of the points in  $A$  and  $B$  respectively;  $\mathbf{R}$  the set of real numbers.

We want to match the stars from the input image with the stars from the catalogue in order to estimate their mutual correspondence and align more precisely their coordinates, including the image centre. This alignment can be achieved by moving the stars from the input image uniformly to optimally adjust them to the corresponding stars from the catalogue. Thus we consider two functions  $\mu$  and  $\lambda$  as defined below:

$$\mu : A \rightarrow B, \quad \mu(a) = b, \quad (2)$$

$$\lambda = \lambda(R, t) : A \rightarrow E^2, \quad \lambda(a | R, t) = Ra - t, \quad (3)$$

where  $a \in A$ ,  $b \in B$  are arbitrary elements of  $A$  and  $B$ ,  $t \in E^2$  is an arbitrary translation vector, and  $R$  is a [2x2] rotation matrix, i.e.  $\det(R) = 1$ ,  $RR^T = I$ ,  $I$  the [2x2] unit matrix. We measure the adjustment between the two point sets using the sum  $S$  of squared distances between corresponding points:

$$S = \sum_a \|\lambda(a) - \mu(a)\|^2, \quad (4)$$

which we are interested to minimize. Thus, our problem is to find such  $\lambda$  and  $\mu$  which satisfy the criterion:

$$\min S = \min_{\lambda, \mu} \sum_a \|\lambda(a) - \mu(a)\|^2 \quad (5)$$

## WEIGHTED ITERATIVE CLOSEST POINT ALGORITHM

Since the introduction of the basic concepts of ICP algorithm by Chen [3] and Besl [2], there have been developed multiple variants of the algorithm, a classification of which can be found in Rusinkiewicz [6]. An analytical solution finding an optimal rotation matrix  $R$  and a translation vector  $t$  is developed by Arun [1]. The implementation of the algorithm is based on the work of the authors above mentioned with the optimization improvements by Kapoutsis [5].

Basically ICP algorithm is expressed by the following sequence:

1. Initialization:  $\lambda : (R, t) = (I, 0)$
2. Matching:  $\mu := \text{Arg min}_{\mu} \sum_a \|\lambda(a) - \mu(a)\|^2$
3. Transforming:  $\lambda := \text{Arg min}_{\lambda} \sum_a \|\lambda(a) - \mu(a)\|^2$
4. Loop to step 2 until convergence or exceeding the iterations limit.

Involving the point weights we change the way the points map to each other. When 2 points have similar assigned weights they will map with greater priority, but without losing the Euclidian distance influence. We want the distance between the points to increase with an amount reflecting their weight difference and to be exactly the Euclidian distance when their weights are the same. This consideration leads us to the following new criteria:

$$S_{new} = \sum_a \left[ \frac{\max(W_A(a), W_B(\mu(a)))}{\min(W_A(a), W_B(\mu(a)))} \right]^2 \|\lambda(a) - \mu(a)\|^2 \quad (6)$$

We evaluate the convergence at step 4 by the minimum of  $S_{new}$ .

The correction coefficient of the norm in (6) is simply a division of the biggest weight with the smallest one that results to a value equal or greater than 1. Only the matching  $\mu$  is affected by this change, and the previous algorithmic steps above mentioned remain the same [1,3,4,6,7].

A crucial drawback of the ICP algorithm is the high computational complexity of the matching operator  $\mu$ . Kapoutsis, Vavoulidis and Pitas [6] propose a morphological ICP algorithm solving the problem for the 3D space by using an array representing the 3D volume by a Voronoi tessellation method on the model points within that volume. Then, the ICP algorithm is employed, where the distance calculations are substituted by simple array references. The complete algorithm is thoroughly described in [6]. In contrast to the  $\mu$  operator,  $\lambda$  can be computed directly, as follows:

Let  $\bar{a}$  and  $\bar{b}$  be the centroids of the respective point sets:

$$\bar{a} = \frac{1}{|A|} \sum_{a \in A} a, \quad \bar{b} = \frac{1}{|B|} \sum_{b \in B} b \quad (7)$$

and  $a'$ ,  $b'$  are the respective central coordinates of  $a \in A$  and  $b \in B$ :

$$a' = a - \bar{a}, \quad b' = b - \bar{b} \quad (8)$$

Hence,

$$\sum_a \|\lambda(a) - \mu(a)\|^2 = \sum_{a, b = \mu(a)} \|Ra - t - b\|^2 = \sum_{a, b = \mu(a)} \|(Ra' - b') + (R\bar{a} - \bar{b} - t)\|^2. \quad (9)$$

Using triangular inequality we can prove that the minimum of (9) is reached at  $(R\bar{a} - \bar{b} - t) = 0$ , i.e. for  $t$  we have:

$$t = R\bar{a} - \bar{b}. \quad (10)$$

Thus, the minimization of (4) is reduced to minimization of  $\sum_a \|Ra' - b'\|^2$  instead of  $\sum_a \|\lambda(a) - \mu(a)\|^2$ . Hence,

$$\begin{aligned} \sum_a \|Ra' - b'\|^2 &= \sum_a (Ra' - b')^T (Ra' - b') = \\ &= \sum_a (a'^T R^T Ra' + b'^T b' - a'^T R^T b' - b'^T Ra') = \\ &= \sum_a \|a'\|^2 + \sum_a \|b'\|^2 - 2\text{Trace}(RH), \end{aligned} \quad (11)$$

where

$$a'^T R^T b' = b'^T Ra' = \text{Trace}(R(a'b'^T)) \quad (12)$$

and

$$H = \sum_a a'b'^T. \quad (13)$$

Hence the problem of minimizing (4) is reduced to finding:

$$\max_R \text{Trace}(RH). \quad (14)$$

Arun [1] proves that (12) is maximized by

$$R = VU^T \quad (15)$$

where

$$H = U\Sigma V^T \quad (16)$$

is the singular value decomposition of  $H$ .

## EXPERIMENTAL RESULTS

For the experiment we are using scanned wide-field photographic plate with 1975 extracted stars up to stellar magnitude 14. All stars are rotated by  $15^\circ$  and translated by  $t = [0.2, 0.2]$  in normalized space  $[1 \times 1]$  resulting to the set  $A$  (see Fig. 1a). In this way translational and rotational noise is simulated. Note that translational and rotational invariance is supported by the proposed method, but not scaling.

If  $B$  is the initial star map we want to align the set  $A$  over set  $B$ .

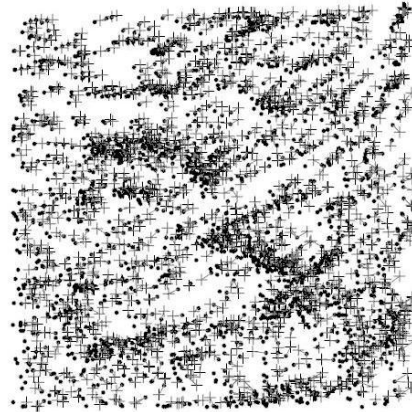
First we are running the simulation without taking in consideration the stars magnitudes. The result of execution is shown on Fig. 1, where convergence to the desired match is achieved in 51 iterations.

The set  $A$  is marked with black crosses and set  $B$  with black dots that help to better visibility of initial and of the final stages of processing illustrated on Fi.1 and 2. Figures 1(a-c) represent the initial, intermediate and final ICP stage, while Fig.1d shows the monotonically decreasing distance between the sets at each stage. The distance at  $y$ -axis

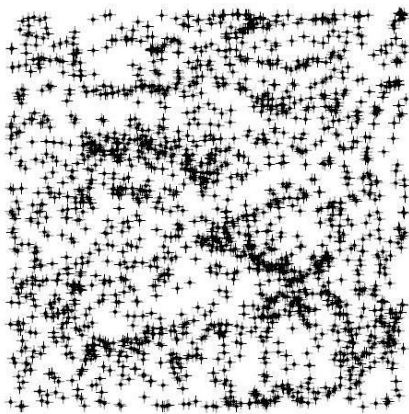
is calculated as sum of all point pairs distances determined by  $\mu_i$  (the minimal  $\mu$  at iteration  $i$ ).



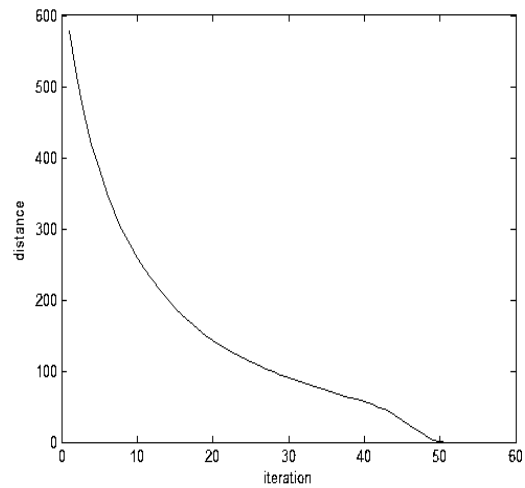
a) *Initial stage*



b) *Intermediate stage*



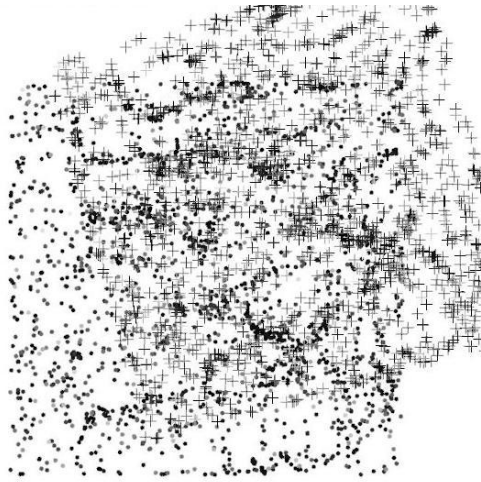
c) *Final stage*



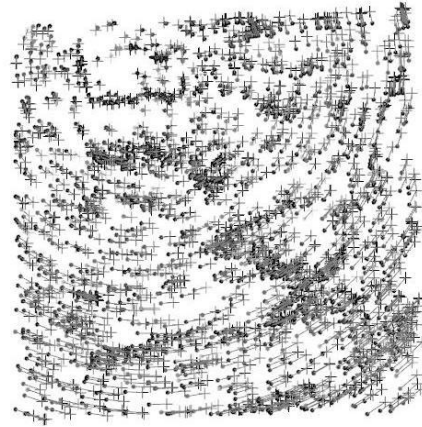
d) *Set distance at each iteration*

**Fig. 1 ICP algorithm process aligning sky maps without using stellar magnitudes.**

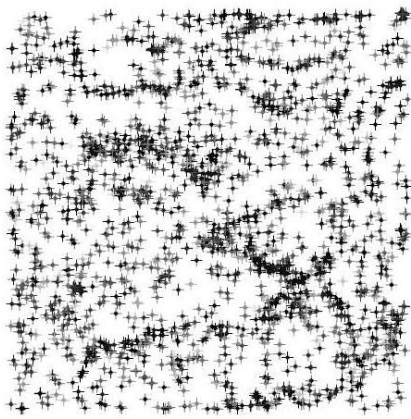
We run the same simulation by involving the star magnitudes as point weights. The result of the process is presented on Fig. 2. The notes from the previous experiment are still valid, except that the marks are in greyscale intensities proportional to the star magnitude, where the brighter stars are marked with darker colours. This time convergence is achieved in 30 iterations which is remarkable improvement.



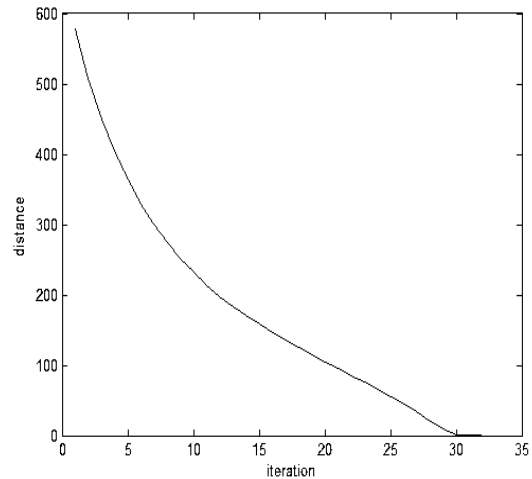
a) *Initial stage*



b) *Intermediate stage*



c) *Final stage*



d) *Maps distance at each iteration*

**Fig. 2 ICP algorithm process aligning sky maps with given stellar magnitudes.**

## CONCLUSION

We illustrated the Iterative Closest Point algorithm applied to the problem of aligning stars in astronomical images by involving the star magnitudes. Two experiments are presented which demonstrate remarkable improvement in algorithm performance when the point sets are supplied with weights. In the common case ICP converges to local minima, which is not necessarily the best coverage. But the provision of more information (e.g. point weights) for calculating point distances decreases this risk.

The author is foreseeing as future work to research on ICP extension considering existing scaling factor between the two point sets.

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## ABOUT THE AUTHORS

Alexander Marinov, Ph.D. Student, Institute of Information and Communication Technologies, Bulgarian Academy of Sciences, Sofia, Phone: +359 2 877 26 75, E-mail: [amarinov@iinf.bas.bg](mailto:amarinov@iinf.bas.bg).

Nadezhda Zlateva, Ph.D. Student, Institute of Information and Communication Technologies, Bulgarian Academy of Sciences, Sofia, Phone: +359 2 877 26 75, E-mail: [nzlateva@iinf.bas.bg](mailto:nzlateva@iinf.bas.bg).

Dimo Dimov, Assoc. Prof., Ph.D., Institute of Information and Communication Technologies, Bulgarian Academy of Sciences, Tel: +359 2 870 64 93, E-mail: [dtdim@iinf.bas.bg](mailto:dtdim@iinf.bas.bg)

Delian Marinov, MSc. on Mathematics, Tel: +359 2 877 26 75.